Matthias Himmelmann

PhD Student of Myfanwy Evans
University of Potsdam
himmelmann1@uni-potsdam.de

SFB TRR 109 Discretization in Geometry and Dynamics

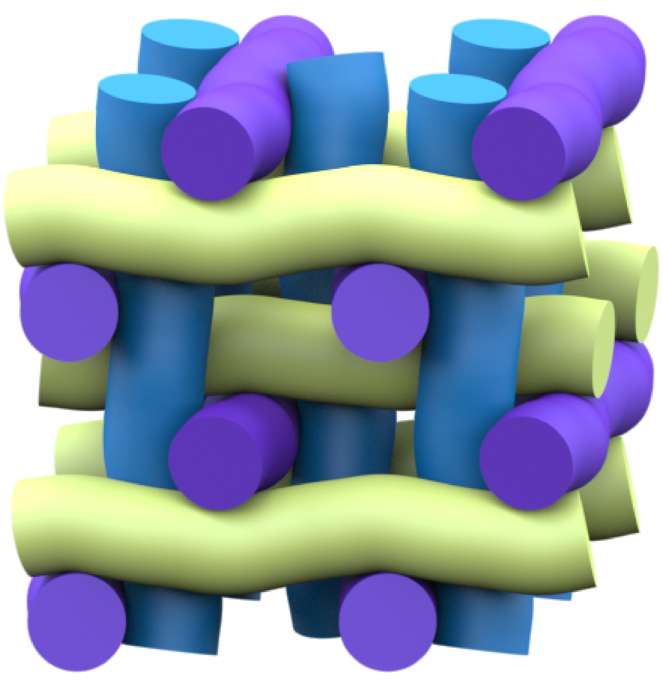
Towards a Robust Tensegrity Model for the Mechanics of Filament Packings

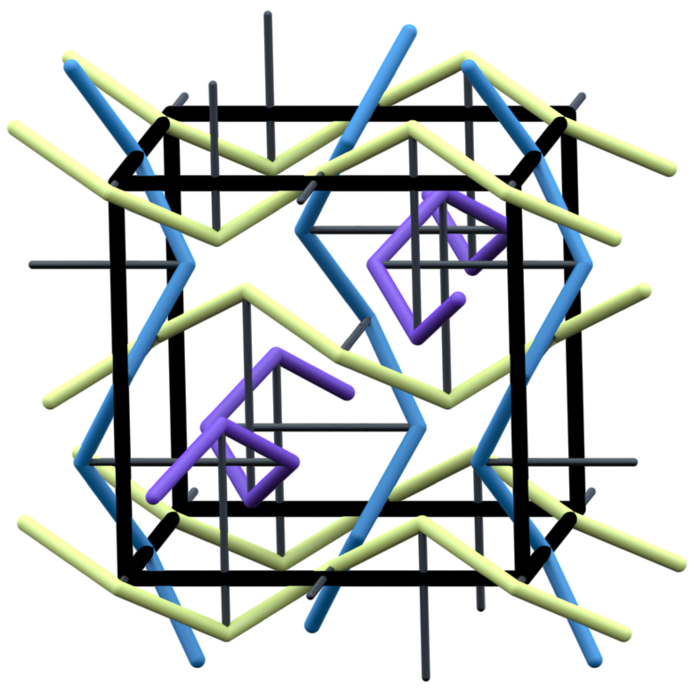
Introduction

My goal is to use modeling and (numerical) algebraic geometry to understand material deformations, particularly in filament packings.

Previous Modeling

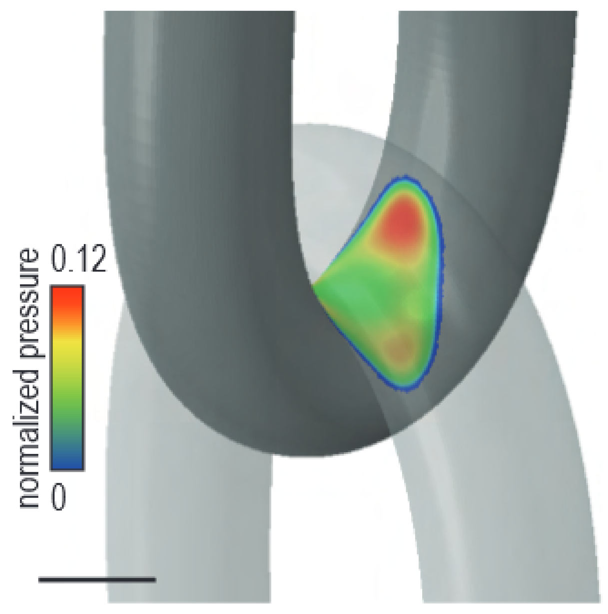
Similar to sphere packings [4], rod packings can be modeled by a tensegrity framework with cables along the rods and a bar for each contact point [5]. Below, the crystal β-Mn is displayed (left, [5]) with its tensegrity representation (right, [5]).



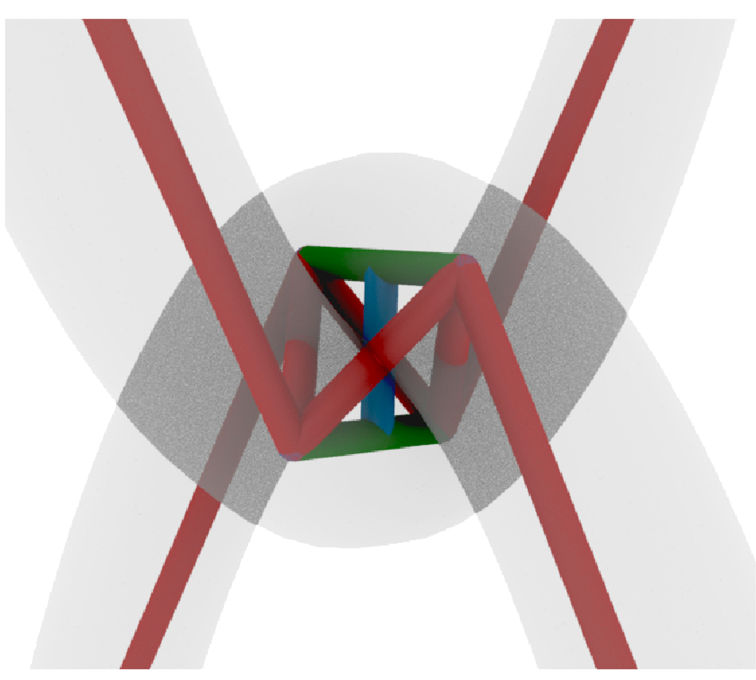


The Orthogonal Clasp

Imagining two tubes around circular arcs in perpendicular planes gives rise to the orthogonal clasp. In practice, this corresponds to two elastic elements wrapping around each other, with forces pulling in opposite directions. The idealized model produces a 1-dimensional contact curve. In fact, it is a loop with four cusps [1,2]. However, experiments suggest there is a 2-dimensional contact patch, with regions of differing pressure (r. [2]). Notably, the maxima are obtained in the idealized curve's cusps.



The Tetrahedral Model



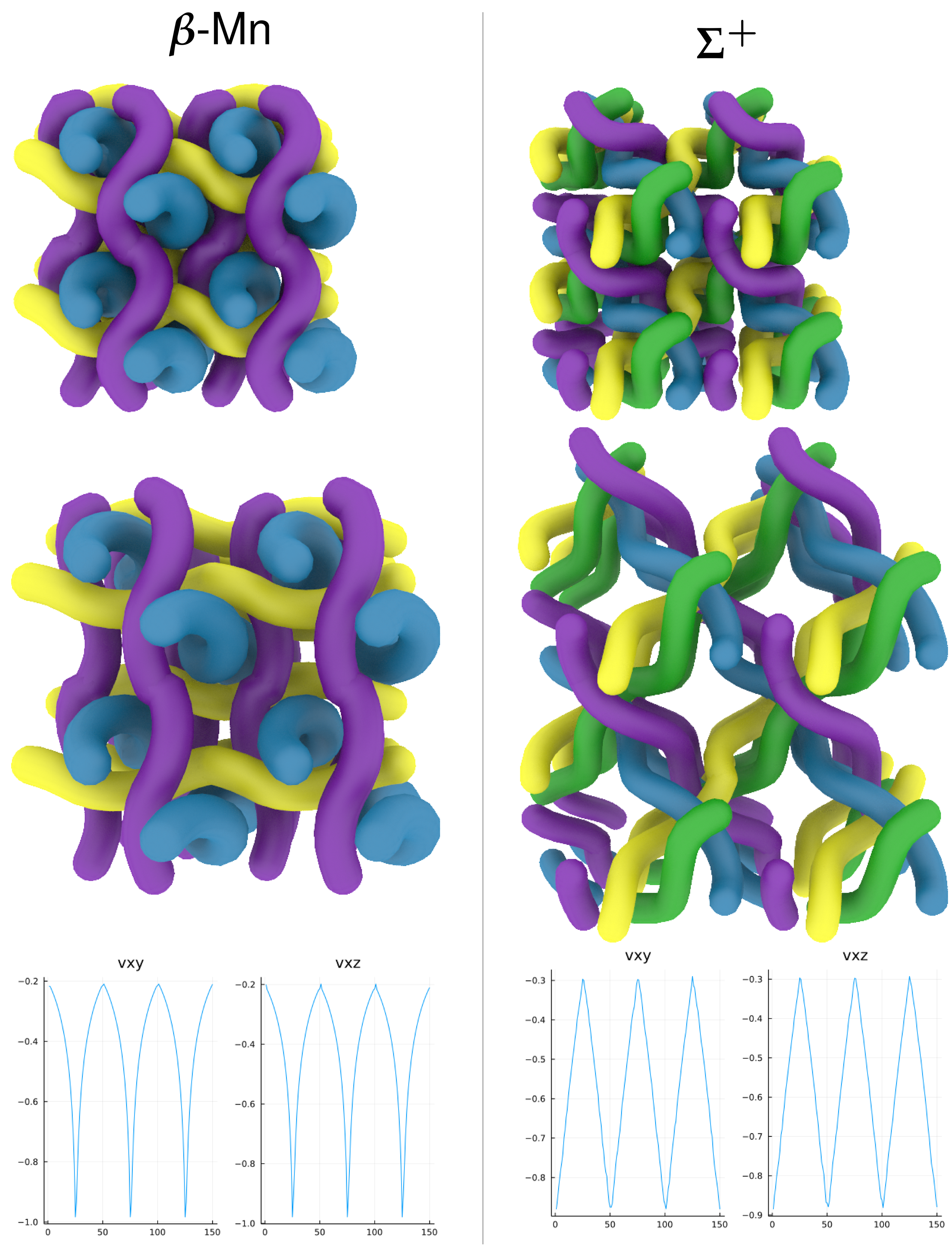
Motivated by the orthogonal clasp's properties, we take a closer look at the contacts between adjacent rods. There are four disconnected regions of maximal pressure. As a model, a tetrahedron is placed inside the rods, with an edge passing through each contact region.

- Analogous to previous models, a chain of cables is placed at the center of each rod (red).
- As the size of the contact surface depends on the applied forces [2], two variable-length bars (green) are introduced. They connecting the cables passing through each rod.
- An orthogonal center bar (blue) with length equal to twice the rods' radii is placed between the two variable-length bars.
- Twisting – an asymmetrization of the contacts – is penalized by the introduction of flexible cables (red) between the vertices.

Polynomial Optimization

The Tetrahedral Model can be expressed as a nonlinear optimization problem with polynomial constraints. If the initial configuration is close enough to the equilibrium, Newton's method converges quadratically to it. In practice, this property is seldomly satisfied. The implementation of the Riemannian Optimization package `HomotopyOpt.jl` [6] for algebraic constraint sets tackles this issue. By using homotopy continuation to approximate the exponential map, encouraging theoretical and practical properties emerge, such as guaranteed convergence, numerical robustness and constituting a general-scope Riemannian Optimization tool.

The Auxetic Filament Packings β-Mn and Σ^+



Outlook

- The mechanics of more complicated (auxetic) rod packings should also be manageable with this model.
- It is possible to incorporate sliding of the rods' contact points in the model by leaving the cables' resting length to be variable.
- While weavings and non-auxetic frameworks seem like the natural way of applying this model, the introduction of repulsive forces (*struts*) to this tensegrity model seems necessary to avoid the collaps of the cables to their natural resting length.

[1] Cantarella et al. Criticality for the Gehring Link Problem. Geom. Topol., 10, 2006.
[2] Grandgeorge et al. Mechanics of two filaments in tight contact. P. Nt. Acd. Sc., 118(15), 2021.
[3] Borcea and Streinu. Geometric auxetics. P. Roy. So. A: Mthm., Phys. Eng. Sc., 471, 2015.
[4] Connelly et al. Ball Packings with Periodic Constraints. Disc. Comp. Geom., 52, 2014.
[5] Oster et al. Re-entrant tensegrity: A 3-periodic, chiral, tensegrity structure that is auxetic. Sc. Adv. 7, 2021.
[6] Heaton and Himmelmann. Euclidean distance and ML retractions by homotopy continuation. Preprint, 2022.